Partial Force Control of Constrained Floating-Base Robots

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Abstract—We present a new method to control a subset of the contact forces exerted by a humanoid robot on the environment. Humanoids are typically in contact with the environment at multiple locations, hence their motion is constrained by contacts. We consider here rigid contacts only. This work is based on the technique of constraint nullspace projection, which proved effective in controlling constrained underactuated robots. However, this technique does not allow to control the constraint forces, so it may fail in certain situations. For instance, every time there is a transition of the contact state (i.e. the robot makes/breaks a contact), failing to control the transient contact force could produce jerky motion. Our idea is to project the system dynamics into the nullspace of a subset of the constraints, so that the remaining constraints can be controlled. This new control law is simple and has low computational cost because it does not require the computation of the mass matrix. Simulations were carried out to validate the presented approach.

I. INTRODUCTION

The control of floating-base robots is challenging because these systems are underactuated, hence they cannot be feedback-linearized [5]. The problem becomes even more complex when the robot is constrained, which is the typical case for humanoids in rigid contact with the ground/environment. Sentis [4] and Park [2] presented a control framework for prioritized motion and force control of constrained humanoids.

Righetti et al. proposed an alternative approach [3] that results in a simpler and more efficient control law. They projected the robot dynamics into the nullspace of the constraints with a kinematic projector, and used the projected equation to derive a control law. The constraint forces are cancelled by the projection, so there is no need to measure them using noisy force measurements.

This approach is safe as long as the contact geometry does not allow the robot to apply unbounded contact forces. If a robot is in contact with the ground only, the forces are limited by its weight, so we can safely not control them. On the contrary, if a robot makes additional contacts, then it can apply much higher forces, which may be only limited by its motor power. In these situations we need to control — besides the motion — the constraint forces too. We can distinguish two cases: either we want to control all the constraint forces, or we want to control a subset of them. This paper deals with the latter case.

II. METHOD

Consider a floating-base robot with n joints that is subject to a set of k nonlinear equality constraints: $c(q, \dot{q}, t) = 0$. Its equations of motion are:

$$M(q)\ddot{q} + h(q,\dot{q}) - J_c(q)^T f = S^T \tau,$$
(1)

where $M \in \mathbb{R}^{n+6 \times n+6}$ is the inertia matrix, $\ddot{q} \in \mathbb{R}^{n+6}$ are the joint accelerations, $h \in \mathbb{R}^{n+6}$ are the bias forces, $S \in \mathbb{R}^{n \times n+6}$ is the matrix selecting the *n* actuated joints, $\tau \in \mathbb{R}^n$ are the joint torques, $J_c = \frac{\partial c}{\partial q} \in \mathbb{R}^{k \times n+6}$ is the constraint Jacobian, and $f \in \mathbb{R}^k$ are the constraint forces. Let us split the constraints in two subsets: the controlled constraints (with Jacobian $J_f \in \mathbb{R}^{k_f \times n+6}$ and forces $f_f \in \mathbb{R}^{k_f}$), and the supporting constraints (with Jacobian $J_s \in \mathbb{R}^{k_s \times n+6}$ and forces $f_s \in \mathbb{R}^{k_s}$), so that:

$$J_c^T = \begin{bmatrix} J_f^T & J_s^T \end{bmatrix}, \qquad f^T = \begin{bmatrix} f_f^T & f_s^T \end{bmatrix}$$

The problem of regulating f_f to a desired value f_f^* may be formulated as:

$$\begin{aligned} \tau^* &= \operatorname*{argmin}_{\tau \in \mathbb{R}^n} ||f_f - f_f^*||^2 \\ s.t. \quad M\ddot{q} + h - J_s^T f_s - J_f^T f_f = S^T \tau \\ J_c \ddot{q} &= -\dot{J}_c \dot{q} \end{aligned}$$

Defining $N_s = I - J_s^+ J_s$, under the assumption that $N_s S^T$ is full-rank (which in practice is true any time $k_s \ge 6$), the infinite solutions of this problem can be expressed as:

$$\tau^* = (N_s^T S^T)^+ N_s^T (-J_f^T f_f^* + M(-J_c^+ \dot{J}_c \dot{q} + N_c \ddot{q}_0) + h),$$

where $N_c = I - J_c^+ J_c$, and $\ddot{q}_0 \in \mathbb{R}^{n+6}$ is an arbitrary joint acceleration vector. Consider now the general case in which the robot has to perform also N - 1 motion tasks. The force control task has highest priority, because it is a physical constraint that cannot be violated by definition. The motion task *i* is defined by a Jacobian J_i and a reference acceleration \ddot{x}_i^* . The control torques can be computed as:

$$\tau^* = (N_s^T S^T)^+ N_s^T (M\ddot{q}_1 + h - J_f^T f_f^*)$$

$$\ddot{q}_i = \ddot{q}_{i+1} + (J_i N_{p(i)})^+ (\ddot{x}_i^* - \dot{J}_i \dot{q} - J_i \ddot{q}_{i+1}) \qquad \forall i \in [1, N]$$

$$N_{p(i)} = N_{p(i+1)} - (J_{i+1} N_{p(i+1)})^+ J_{i+1} N_{p(i+1)},$$

(2)

where $J_N = J_c$, $N_{p(N)} = I$, $\ddot{x}_N^* = 0$ and $\ddot{q}_{N+1} = 0$. Kinematics and dynamics are decoupled, so τ^* can be efficiently computed with the Recursive Newton-Euler Algorithm.

III. TESTS

We tested our approach on a customized version of the Compliant huManoid (CoMan) simulator [1]. The robot has 23 DoFs: 4 in each arm, 3 in the torso and 6 in each leg. Direct and inverse dynamics, were computed with C functions, generated with Robotran [6]. Contact forces were simulated using spring-damper models with friction (stiffness $2 \cdot 10^5 N/m$ and damping $10^3 Ns/m$).

A. Test 1

In this test the robot made contact with a rigid wall using its right hand, and it regulated the contact force to 20 N. The



Fig. 1: Test 1: the robot made contact on the yellow wall; then it moved its COM towards the wall and back.

contact forces at the feet were not controlled. After making contact, we shifted the desired position of the COM along the y direction, so that the robot leaned against the wall, exploiting the additional support provided by the contact on its hand. Overall, the robot performed three tasks, listed here with decreasing priority: right hand force control, 1 DoF (wall normal direction); COM position control, 2 DoFs (xy plane); posture control, 29 DoFs. The RMSE for the force task was about 0.01 N, while for the COM task it was about 0.6 mm.

B. Test 2

This test tackles the switching between double and single support in walking. These hard switches cause discontinuities in the control action, which may result in jerky motion or instability. We show how partial force control can eliminate these discontinuities. When switching from double to single support, we control the forces associated to the removed constraints, to generate a smooth transition of that force to zero. After moving the COM on the left foot, we switched to single support, removing the right foot constraints. To avoid a discontinuity in the right foot force, we activated a (partial)



Fig. 2: Test 2: force on the right foot (normal direction).

force control, which smoothly regulated the force towards zero. At this point the foot was ready to be moved, but we did not move it, because we wanted to switch back to double support. We increased the contact force at the right foot to 100 N, and then we switched the controller back to double support, reintroducing the right foot constraints. This ensured a smooth transition between constraint states (note the absence of discontinuities in the contact force in Fig. 2). Overall, the robot performed three tasks, listed here with decreasing priority: right foot force control, 3 DoF; COM position control, 2 DoF (xy plane); posture control, 29 DoF. The RMSEs were negligible for both tasks.

IV. CONCLUSIONS

We presented a new control law for force control of constrained floating-base systems. This control law extends the technique of constraint nullspace projection, to allow the regulation of a subset of the constraint forces exerted by the robot. Under the assumption of leaving at least 6 constraints for the nullspace projection (e.g. not controlling the forces exerted by at least one foot), the control law takes a particularly simple form, which results in an efficient computation that does not require the mass matrix calculation (computation time below 1 ms on a 2.83 GHz CPU).

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